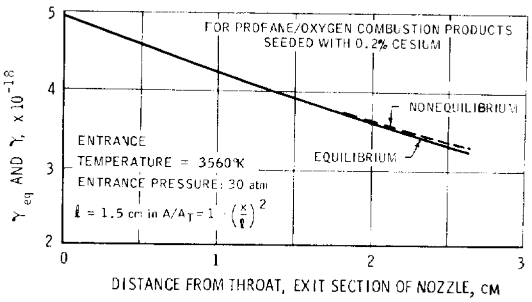

Fig. 1 Comparison of equilibrium and nonequilibrium ionization in expanded flow through an axisymmetric, hyperbolic nozzle

Here γ and γ_{eq} are, respectively, nonequilibrium and equilibrium electron densities divided by gas density, and α is the recombination coefficient for cesium ions and electrons. A value of rate coefficient $\alpha = 10^{-7}(250/T)^{3/2}$ was used. This is the lowest value of rate coefficient given by Eschenroeder and, accordingly, produces maximum ionization nonequilibrium. Values of $(\gamma_{eq})_x$ were obtained from the Saha equation for degree of ionization as a function of temperature and pressure.

Figure 1 compares γ and γ_{eq} as obtained. Ionization due to nonequilibrium is negligible under conditions that purposely were taken as extreme for continuous MHD generator operation. Little or no benefit is to be derived from this type of ionization nonequilibrium in present-day MHD generator design.

Reference

[1] Eschenroeder, A. Q., "Ionization nonequilibrium in expanding flows," ARS J. **32**, 196–203 (1962).

Concave Surfaces in Free Molecule Flow

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Nomenclature

- m = molecular mass
- N_i = total incident molecular flux
- N_b = incident molecular flux from multiple reflections
- N_∞ = incident molecular flux from freestream
- r = radius of cylindrical arc surface
- r_{12} = separation of surface elements $d\Sigma_1$, $d\Sigma_2$
- R = gas constant (per unit mass)
- S = molecular speed ratio = $U_\infty/(2RT_\infty)^{1/2}$
- T_∞ = freestream temperature
- T_b = temperature of surface
- U_∞ = freestream velocity
- z = spanwise coordinate
- α = angle of incidence
- θ = polar coordinate
- Θ = limit on θ
- ρ_∞ = freestream density

In two recent papers Chahine[1, 2] has studied the aerodynamic characteristics of surfaces with cylindrical and spherical curvature, concave to a hyperthermal free molecule flow. It is the purpose of this note to point out that Chahine's results for the drag of the cylindrical surface are in error, and to give the corrections. An alternative approach to the problem used by the present author[3] has proved advantageous in the axisymmetric case in that the numerical evaluation of the results is relatively straightforward, whereas Chahine's results are left in the form of intractable quadruple integrals.

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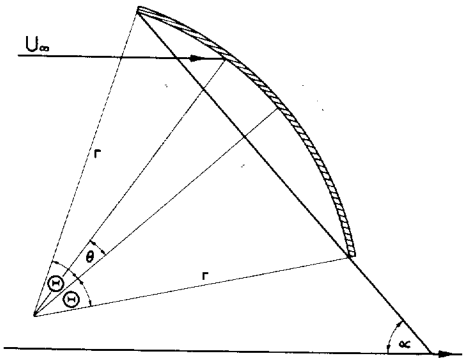

Fig. 1 Geometry of cylindrical surface

Following Chahine, consider an arc, infinite in extent, of a circular cylindrical surface (Fig. 1). The generators of the surface are normal to a free molecule flow, and no part of the surface is shielded from the freestream. In this paper the mean thermal velocity of the gas molecules in comparison with the freestream velocity is neglected, and a perfectly diffuse reflection of molecules from the surface, with perfect thermal accommodation, is assumed.

The momentum exchange normal to the surface per unit area per unit time may be expressed as

$$p(\theta) = p_\infty(\theta) + p_b(\theta) + p_r(\theta) \tag{1}$$

where p_∞ is the pressure component due to incident freestream molecules, p_b is that due to multiple reflections of molecules from other parts of the surface, and p_r is the component due to re-emission of molecules from the surface. Similarly, the tangential momentum exchange is

$$\tau(\theta) = \tau_\infty(\theta) + \tau_b(\theta) \tag{2}$$

Since re-emission is diffuse, $\tau_r = 0$. The freestream and re-emission terms take the well-known forms[4]

$$p_\infty(\theta) = \rho_\infty U_\infty^2 \sin^2(\alpha - \theta) \tag{3}$$

$$p_r(\theta) = mN_i(\theta)(\tfrac{1}{2}\pi RT_b)^{1/2} \tag{4}$$

$$\tau_\infty(\theta) = \rho_\infty U_\infty^2 \sin(\alpha - \theta)\cos(\alpha - \theta) \tag{5}$$

The multiple reflection terms remain to be determined.

Integration of the normal momentum components of the re-emitted molecules over all possible directions of re-emission shows that Eq. (4) is compatible with a mean velocity of re-emission given by

$$c = \tfrac{3}{4}(2\pi RT_b)^{1/2} \tag{6}$$

where the cosine law of diffuse reflection is taken to hold. In order to obtain the multiple reflection terms, one assumes that the molecules are all emitted from the surface with this mean velocity. From the geometry of Fig. 2 one finds that the number of molecules emitted by a surface element $d\Sigma_2$

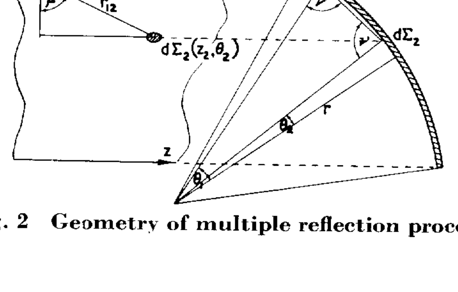

Fig. 2 Geometry of multiple reflection process

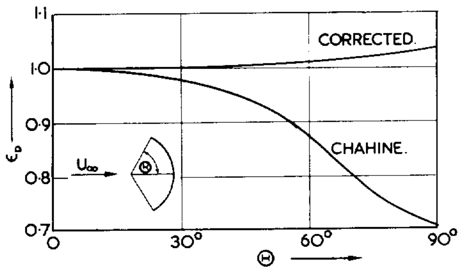

Fig. 3 Values of ϵ_D for cylindrical surface

and intercepted by $d\Sigma_1$ in unit time is

$$dN_b(\theta_1)\, d\Sigma_1\, (\theta_1,z_1) = \frac{\cos\delta_1 \cos\delta_2}{\pi r_{12}^2} N_i(\theta_2)\, d\Sigma_2\, (\theta_2,z_2)\, d\Sigma_1\, (\theta_1,z_1) \quad (7)$$

where the cosine law of re-emission again is used. Here δ_1 and δ_2 are the angles between the normals at $d\Sigma_1$ and $d\Sigma_2$, respectively, and the path traversed by the molecules. The consequent pressure arising at $d\Sigma_1$ is

$$dp_b(\theta_1) = mc \frac{\cos^2\delta_1 \cos\delta_2}{\pi r_{12}^2} N_i(\theta_2)\, r\, d\theta_2\, dz_2 \quad (8)$$

The corresponding tangential shear component normal to the generators is

$$d\tau_b(\theta_1) = -mc \frac{\cos\delta_1 \cos\delta_2}{\pi r_{12}^2} \cos\mu \sin\nu\, N_i(\theta_2)\, r\, d\theta_2\, dz_2 \quad (9)$$

where μ and ν are defined in Fig. 2. One is led to

$$dp_b(\theta_1) = \frac{1}{\pi} mc\, N_i(\theta_2) \times \frac{8r^4 \sin^6\frac{1}{2}(\theta_1 - \theta_2)}{[(z_1 - z_2)^2 + 4r^2 \sin^2\frac{1}{2}(\theta_1 - \theta_2)]^{5/2}}\, d\theta_2 dz_2 \quad (10)$$

$$d\tau_b(\theta_1) = -\frac{1}{\pi} mc\, N_i(\theta_2) \times \frac{8r^4 \sin^5\frac{1}{2}(\theta_1 - \theta_2) \cos\frac{1}{2}(\theta_1 - \theta_2)}{[(z_1 - z_2)^2 + 4r^2 \sin^2\frac{1}{2}(\theta_1 - \theta_2)]^{5/2}}\, d\theta_2 dz_2 \quad (11)$$

Integration with respect to z_2 from $-\infty$ to $+\infty$ and substitution for c from Eq. (6) now yields

$$p_b(\theta_1) = \frac{m}{2\pi} (2\pi R T_b)^{1/2} \int_{-\Theta}^{\Theta} N_i(\theta_2) \sin^2\frac{1}{2}(\theta_1 - \theta_2)\, d\theta_2 \quad (12)$$

$$\tau_b(\theta_1) = -\frac{m}{2\pi} (2\pi R T_b)^{1/2} \int_{-\Theta}^{\Theta} N_i(\theta_2) \times \sin\frac{1}{2}(\theta_1 - \theta_2) \cos\frac{1}{2}(\theta_1 - \theta_2)\, d\theta_2 \quad (13)$$

In place of Eq. (13), however, Chahine quotes

$$\tau_b(\theta_1) = -\frac{m}{2\pi} (2\pi R T_b)^{1/2} \int_{-\Theta}^{\Theta} N_i(\theta_2) \sin(\theta_1 - \theta_2)\, d\theta_2 \quad (13a)$$

which is greater by a factor of 2. As will be seen presently, Chahine's use of the latter equation has given rise to misleading results.

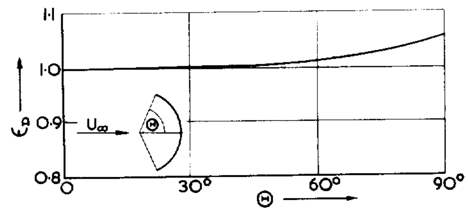

Fig. 4 Values of ϵ_D for spherical surface

The lift and drag of the surface follow from

$$L = r \int_{-\Theta}^{\Theta} [p(\theta_1) \cos(\alpha - \theta_1) - \tau(\theta_1) \sin(\alpha - \theta_1)]\, d\theta_1 \quad (14)$$

$$D = r \int_{-\Theta}^{\Theta} [p(\theta_1) \sin(\alpha - \theta_1) + \tau(\theta_1) \cos(\alpha - \theta_1)]\, d\theta_1 \quad (15)$$

and can be evaluated once $N_i(\theta_1)$ is known. N_i is determined by the geometry of the surface and in this case is the solution of the integral equation

$$N_i(\theta_1) = N_\infty(\theta_1) + \frac{1}{4} \int_{-\Theta}^{\Theta} N_i(\theta_2) \sin\frac{1}{2}|\theta_1 - \theta_2|\, d\theta_2 \quad (16)$$

as is shown by Chahine.

The effect of surface concavity may be demonstrated conveniently by writing the aerodynamic coefficients (nondimensionalized with respect to the chord) in the forms

$$C_D = 2 \sin\alpha + \epsilon_D(\Theta,\alpha)(\pi^{1/2}/S)(T_b/T_\infty)^{1/2} \sin^2\alpha \quad (17)$$

$$C_L = \epsilon_L(\Theta,\alpha)(\pi^{1/2}/S)(T_b/T_\infty)^{1/2} \sin\alpha \cos\alpha \quad (18)$$

For $\Theta = 0$, $\epsilon_D = \epsilon_L = 1$, and the expressions reduce to previously known results for a flat plate at incidence α.

As an example, consider the case where $\alpha = \pi/2$ and $0 \leqslant \Theta \leqslant (\pi/2)$. Then $C_L = 0$ and

$$C_D = 2 + \epsilon_D(\Theta)(\pi^{1/2}/S)(T_b/T_\infty)^{1/2} \quad (19)$$

Figure 3 shows the variation of ϵ_D with Θ for this case and illustrates the significance of the error incurred by Chahine in omitting the factor $\frac{1}{2}$ in the expression for τ_b [Eq. (13a)]. Since τ_b gives rise to a significant negative drag contribution, Chahine's values are too low. The use of the correct expression for τ_b [Eq. (13)], in fact, leads to a complete reversal of the trend of ϵ_D with increasing concavity. As Θ increases, ϵ_D becomes progressively not less than the flat plate value, as Chahine's results indicate, but greater.

Chahine has given no numerical results for the drag of the spherical surface, but the values of ϵ_D for this surface with $\alpha = \pi/2$, as calculated by the method of Ref. 3, are shown in Fig. 4.

References

[1] Chahine, M. T., *Proceedings of the 11th Astronautical Congress, Stockholm 1960, Main Sessions* (Springer-Verlag, Vienna, 1961), pp. 473–482.

[2] Chahine, M. T., "Rarefied gas dynamics," *Proceedings of the 2nd International Symposium on Rarefied Gas Dynamics*, edited by L. Talbot (Academic Press Inc., New York, 1961), pp. 209–222.

[3] Pratt, M. J., "The free molecule flow characteristics of concave surfaces," D.C.Ae. Thesis, College of Aeronautics, Cranfield, England (1962); unpublished.

[4] Hayes, W. D. and Probstein, R. F., *Hypersonic Flow Theory* (Academic Press Inc., New York, 1959), 1st ed., Chap. X.